COGNITION AND KNOWLEDGE OF THE WORLD

DOMAIN

Montana PreK-12 Mathematics Standards for Mathematical Practice

The Standards for PreK-12 Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with long-standing importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches. Building on the inherent problem-solving abilities of people over time, students can understand that mathematics is relevant when studied in a cultural context that applies to real-world situations and environments.

2. Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions within a cultural context, including those of Montana American Indians. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics.
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. This includes solving problems within a cultural context, including those of Montana American Indians. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or

solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Pre-Kindergarten Mathematics Overview

The preschool/pre-kindergarten population includes children between at least 2 years, 9 months until they are kindergarten eligible. A majority attend programs in diverse settings—community-based early care and education centers, family child care, Head Start, and public preschools. Some children do not attend any formal program. These standards apply to children who are at the end of that age group, meaning older four- and younger five-year olds.

In this age group, foundations of mathematical understanding are formed out of children's experiences with real objects and materials. The standards can be promoted through play and exploration activities, and embedded in almost all daily activities. They should not be limited to "math time." The standards should be considered guideposts to facilitate young children's underlying mathematical understanding.

In preschool or pre-kindergarten, activity time should focus on two critical areas: (1) developing an understanding of whole numbers to 10, including concepts of one-to-one correspondence, counting, cardinality (the number of items in a set), and comparison; (2) recognizing two-dimensional shapes, describing spatial relationships, and sorting and classifying objects by one or more attributes. Relatively more learning time should be devoted to developing children's sense of number as quantity than to other mathematics topics.

(1) These young children begin counting and quantifying numbers up to 10. Children begin with oral counting and recognition of numerals and word names for numbers. Experience with counting naturally leads to quantification. Children count objects and learn that the sizes, shapes, positions, or purposes of objects do not affect the total number of objects in the group. One-to-one correspondence with its matching of elements between the sets, provides the foundation for the comparison of groups and the development of comparative language such as, *more than, less than,* and *equal to*.

(2) Young children explore shapes and the relationships among them. They identify the attributes of different shapes including the length, area, weight by using vocabulary such as: *long, short, tall, heavy, light, big, small, wide, narrow*. They compare objects using comparative language such as: *longer/shorter, same length, heavier/lighter*. They explore and create 2- and 3-dimensional shapes by using various manipulative and play materials such as: popsicle sticks, blocks, pipe cleaners, and pattern blocks. They sort, categorize, and classify objects and identify basic 2-dimensional shapes using the appropriate language.

Counting and Cardinality

- Know number names and the counting sequence.
- Count to tell the number of objects.
- Compare numbers.

Operations and Algebraic Thinking

- Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

Measurement and Data

- Describe and compare measurable attributes.
- Classify objects and count the number of objects in each category.
- Work with money.

Geometry

- Identify and describe shapes (squares, circles, triangles, rectangles).
- Analyze, compare, create, and compose shapes.

Based on the Massachusetts Curriculum Framework for Mathematics, March 2011
by the Massachusetts Department of Elementary and Secondary Education.

Standards for Mathematical Practice: Pre-Kindergarten Explanations and Examples

Standards	***Explanations and Examples***
Students are expected to:	The Standards for Mathematical Practice describe ways in which students ought to engage with the subject matter as they grow in mathematical maturity and expertise.
PK.MP.1. Make sense of problems and persevere in solving them.	Pre-Kindergarten students may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" or they may try another strategy.
PK.MP.2. Reason abstractly and quantitatively.	Pre-Kindergarten students begin to recognize that a number represents a specific quantity. Then, they connect the quantity to written symbols.
PK.MP.3. Construct viable arguments and critique the reasoning of others.	Pre-Kindergarten students construct arguments using concrete referents, such as objects, pictures, drawings, and actions. They also begin to develop their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?"
PK.MP.4. Model with mathematics.	Pre-Kindergarten students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart or list. Students need opportunities to connect the different representations and explain the connections.
PK.MP.5. Use appropriate tools strategically.	Pre-Kindergarten students begin to consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful.
PK.MP.6. Attend to precision.	Pre-Kindergarten students begin to develop their mathematical communication skills.
PK.MP.7. Look for and make use of structure.	Pre-Kindergarten students begin to discern a pattern or structure (i.e., abab patterns).
PK.MP.8. Look for and express regularity in repeated reasoning.	(Begins in kindergarten.)

Mathematics Standards: Counting and Cardinality

Pre-Kindergarten (older 4-year-olds to younger 5-year-olds)	Kindergarten
Know number names and the count sequence	
PK.CC.1. Count to 10 by ones.	K.CC.1. Count to 100 by ones and by tens.
PK.CC.2. (Begins in kindergarten.)	K.CC.2. Count forward beginning from a given number within the known sequence (instead of having to begin at 1).
PK.CC.3. Represent a number of objects by matching to a written numeral 0 – 5 (with 0 representing a count of no objects).	K.CC.3. Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).
Count to tell the number of objects	
PK.CC.4. Understand the relationship between numerals and quantities to 10. a. When counting objects, say the number names in the standards order, pairing each object with one and only one number name and each number name with one and only one object from a variety of cultural contexts, including those of Montana American Indians. b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. c. Understand that each successive number name refers to a quantity that is one larger.	K.CC.4. Understand the relationship between numbers and quantities; connect counting to cardinality. a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object from a variety of cultural contexts, including those of Montana American Indians. b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. c. Understand that each successive number name refers to a quantity that is one larger.
PK.CC.5. Count to answer "how many?" questions about as many as 10 things arranged in a line, a rectangular array, or a circle, or as many as 5 things in a scattered configuration; given a number from 1-10, count out that many objects from a variety of cultural contexts, including those of Montana American Indians.	K.CC.5. Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects from a variety of cultural contexts, including those of Montana American Indians.
Compare numbers	
PK.CC.6. Identify "first" and "last" related to order or position.	K.CC.6. Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.
PK.CC.7. (Begins in kindergarten.)	K.CC.7. Compare two numbers between 1 and 10 presented as written numerals.

Mathematics Standards: Operations and Algebraic Thinking

Pre-Kindergarten (older 4-year-olds to younger 5-year-olds)	Kindergarten
Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from	
PK.OA.1. With support and prompting, demonstrate an understanding of addition and subtraction by using objects, fingers, and responding to practical situations (e.g., if we have 3 apples and add 2 more, how many apples do we have in all?).	K.OA.1. Represent addition and subtraction with objects, fingers, mental images, drawings[2], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.
PK.OA.2. (Begins in kindergarten.)	K.OA.2. Solve addition and subtraction word problems from a variety of cultural contexts, including those of Montana American Indians, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.
PK.OA.3. (Begins in kindergarten.)	K.OA.3. Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).
PK.OA.4. (Begins in kindergarten.)	K.OA.4. For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.
PK.OA.5. (Begins in kindergarten).	K.OA.5. Fluently add and subtract within 5.

Mathematics Standards: Number and Operations in Base Ten

Pre-Kindergarten (older 4-year-olds to younger 5-year-olds)	Kindergarten
Work with numbers 11-19 to gain foundations for place value	
PK.NBT.1. (Begins in kindergarten).	K.NBT.1. Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (such as 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

Mathematics Standards: Measurement and Data

Pre-Kindergarten (older 4-year-olds to younger 5-year-olds)	Kindergarten
Describe and compare measurable attributes	
PK.MD.1. Recognize the attributes of length, area, weight, and capacity of everyday objects using appropriate vocabulary (e.g., *long, short, tall, heavy, light, big, small, wide, narrow*).	K.MD.1. Describe measurable attributes of objects, such as length or weight.
Classify objects and count the number of objects in each category	
PK.MD.2. Compare the attributes of length and weight for two objects, including longer/shorter, same length; heavier/lighter, same weight; holds more/less, holds the same amount.	K.MD.2. Describe several measurable attributes of a single object.
PK.MD.3. Sort, categorize, and classify objects by more than one attribute.	K.MD.3. Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.*
PK.MD.4. (Begins in kindergarten.)	K.MD.4. Classify objects from a variety of cultural contexts, including those of Montana American Indians, into given categories; count the numbers of objects in each category and sort the categories by count.

Mathematics Standards: Geometry

Pre-Kindergarten (older 4-year-olds to younger 5-year-olds)	Kindergarten
Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres)	
PK.G.1. Identify relative position of objects in space, and use appropriate language (e.g., *beside, inside, next to, close to, above, below, apart*).	K.G.1. Describe objects, including those of Montana American Indians, in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above, below, beside, in front of, behind,* and *next to*.
PK.G.2. Identify various two-dimensional shapes regardless of their size.	K.G.2. Correctly name shapes regardless of their orientations or overall size.
PK.G.3. (Begins in kindergarten.)	K.G.3. Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid").
Analyze, compare, create, and compose shapes	
PK.G.4. Analyze, compare, and sort two- and three-dimensional shapes and objects of different sizes, using informal language to describe their similarities, differences, and other attributes (e.g., color, size, shape).	K.G.4. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).
PK.G.5. Create and represent three-dimensional shapes (ball/sphere, square box/cube, tube/cylinder) using various manipulative materials (e.g., popsicle sticks, blocks, pipe cleaners, pattern blocks, clay).	K.G.5. Model shapes in the world from a variety of cultural contexts, including those of Montana American Indians, by building shapes from components (e.g., sticks and clay balls) and drawing shapes.
PK.G.6. (Begins in kindergarten.)	K.G.6. Compose simple shapes to form larger shapes. *For example, "Can you join these two triangles with full sides touching to make a rectangle?"*

Standards for The Arts: Pre-Kindergarten

Note: Standards refer to outcomes by the end of pre-kindergarten.

Standards	***Visual Arts***
1. Expresses oneself and represents what s/he knows, thinks, believes, and feels through visual arts.	a. Experiments with a variety of mediums and methods of using art materials (such as using a big brush to paint broad strokes, combining colors, etc.). b. Shows an interest in what can be created with tools, texture, color, and technique. c. Uses materials to build and create "pieces" that represent another item (blocks become a cast; clay becomes a snake). d. Chooses materials and subjects with intent and purpose. e. Paints, draws, and constructs models based on observations.
2. Responds and reacts to visual arts created by themselves and others.	a. Expresses an interest in drawings, sculptures, models, paintings, and art creations of others. b. Identifies similarities and differences among sample of visual art. c. Shares opinions about visual arts, creations, and experiences.
Standards	***Music***
3. Expresses oneself by engaging in musical activities.	a. Participates with increasing interest and enjoyment in a variety of music activities including listening to music, singing songs, performing finger plays, and experimenting with various musical instruments. b. Engages in singing, making-up silly and rhyming verses, imitating rhythmic patterns, and using music to tell stories and express feelings. c. Engages in music activities having different moods, tempos, and rhythms. d. Creates sounds using a variety of instruments.
4. Responds and reacts during musical activities.	a. Moves and keeps rhythm to different kinds of music. b. Reacts to music through oral, written, or visual expression. c. Compares and contrasts different samples of music. d. Repeats, responds, and/or reacts to lyrics and/or melodies.
Standards	***Theatre/Dramatic Play***
5. Participates in a variety of dramatic play activities to represent fantasy and real life experiences.	a. Represents fantasy, real-life, imagination, and literature through dramatic play. b. Assumes the role of something or someone else and attempts to speak in the appropriate manner and tone. c. Participates in teacher-guided and/or spontaneous dramatic play activities such as acting out a story. d. Uses basic props, and costume pieces to establish time, setting, and character.
Standards	***Dance/Creative Movement***
6. Expresses what s/he knows, thinks, feels, and believes through dance and creative movement.	a. Uses movement to interpret or imitate feelings, animals, and such things as plants growing, or a rainstorm. b. Uses his/her body in a variety of ways (dance, march, hop, jump, sway, clap, snap, stomp, twist, turn, etc.). c. Uses creative movement props such as crepe paper, streamers, hoops, and scarves to create special movements and dances. d. Learns simple, repetitive dance steps and routines. e. Moves in spontaneous and imaginative ways to music, songs, rhythm, and silence.

Standards for Social Studies: Pre-Kindergarten

Note: Standards refer to outcomes by the end of pre-kindergarten.

COMING SOON: New Montana Common Core Standards for Literacy in History/Social Studies

Standards for Science: Pre-Kindergarten

Note: Standards refer to outcomes by the end of pre-kindergarten.

COMING SOON: New Montana Common Core Standards for Literacy in Science and the Next Generation Science Standards